IYSE 6420 Fall 2020 Homework3

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1. Maxwell

**Sample y1, . . . , yn, comes from Maxwell distribution with a density**

**Assume an exponential prior on θ**

**(a) Show that posterior belongs to gamma family and depends on data via**

Using Bayes theorem:

Let

The distribution belongs to gamma family: where ,

Since and 2 are constant, it depends on data via

**(b) For λ = 1/2 and y1 = 1.4,y2 = 3.1, and y3 = 2.5, find Bayes estimator for θ. How the**

**Bayes estimator compares to the MLE and prior mean. The MLE for θ is**

Bayes estimator

MLE

Prior mean

Prior mean > Bayes estimator > MLE, since using Bayes estimator we are taking prior belief into consideration, so the result is between MLE and prior. The result is closer to MLE because is small. If is large it will be closer to prior mean.

**(c) Using MATLAB/Octave/R/Python to calculate the 95% equitailed credible set for θ.**

95% equitailed credible set for θ

(0.20274964145781615, 1.16472100217966)

Python Code

from scipy.stats import gamma

alpha = 5.5

beta = 0.5 \* (1.4\*\*2 + 3.1\*\*2 + 2.5\*\*2 + 1)

gamma(alpha, *scale*=1/beta).interval(0.95)

# (0.20274964145781615, 1.16472100217966)

**(d) Find a prediction for a future single observation. For this, you will need the mean of Maxwell, which is**

A prediction for a future single observation

Python Code

import math

alpha = 5.5

beta = 0.5 \* (1.4\*\*2 + 3.1\*\*2 + 2.5\*\*2 + 1)

rv = gamma(alpha, *scale* = 1/beta)

def mean\_maxwell(*theta*):

return 2 \* (2/math.pi/theta) \*\* (0.5)

rv.expect(mean\_maxwell)

# 2.2444986268225957

2. Jeremy Mixture

**(a) Show that for likelihood and mixture prior**

**the posterior is a mixture of**

**where**

Posterior

**(b) Now we assume and the prior for θ is a mixture**

**Find the posterior and Bayes estimator for θ if X = 98.**

Posterior

from part(a)

If likelihood and mixture prior

Then posterior

Where

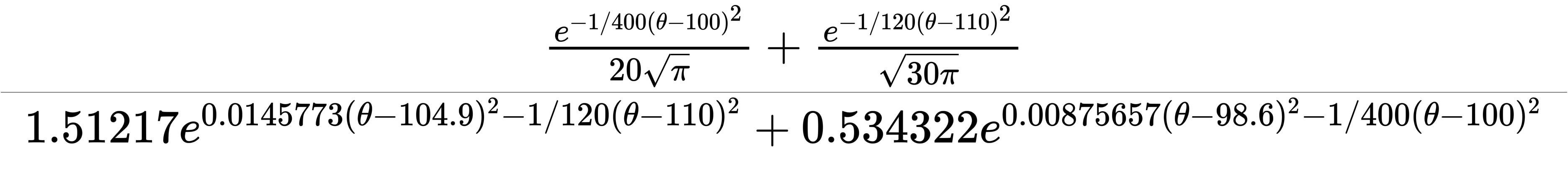
From the equation

If X = 98, follows Normal likelihood + Normal prior

Now we need to calculate to get

So

When



The mean of the posterior is a Bayes estimator of a parameter

3. Mendel’s Experiment with Peas

**For the height trait, Mendel’s model suggests that 3/4 of the plants grown from a cross between tall and short height strains of pea lines will be of the tall height variety. After breeding 1064 of these plants, 787 resulted as the tall height variety. The reasonable model for the number of tall height results from n experiments is binomial Bin(n, p). Complete a Bayesian model with beta Be(12, 4) prior on the unknown proportion p.**

**(a) What are prior and posterior means?**

Binomial Likelihood and Beta Prior

Prior mean

Posterior mean

**(b) Find posterior probability of hypothesis H0 : p ≤ 3/4?**

Posterior

CDF

Python Code

from scipy.stats import beta

a, b = 799, 281

rv = beta(a, b)

rv.cdf(0.75)

# 0.7758595145276612

**(c) Find a 95% equitailed credible set for the true proportion of tall height plants obtained**

**from the given cross.**

95% equitailed credible set

(0.7132478379195061, 0.7655405496526497)

Python Code

from scipy.stats import beta

a, b = 799, 281

rv = beta(a, b)

rv.interval(0.95)

# (0.7132478379195061, 0.7655405496526497)